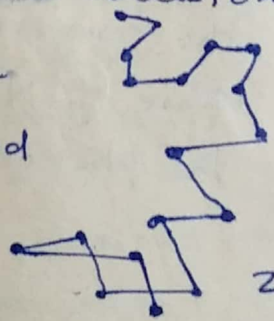
Mean free path and its expression.

A molecule has the finite size and moves in the space of the vessel. It collides with the other molecules and the walls of the containing vessel. The path covered by a molecule between any two consecutive collisions is a straight line and is called the free path. The direction of the molecule is changed after every collision. After a number of collisions, the total path appears to be Zig-Zag and the free path is not constant. Therefore, a term mean free path is used to indicate the mean distance travelled by a molecule between two collisions.

It is defined as the mean distance traversed by a molecule between two successive collisions.

If $\lambda_1, \lambda_2, \lambda_3, ------, \lambda_n$ are the successive free paths traversed and N is the total number of collisions suffered.



Zig-Zag path

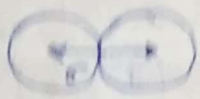
then $$\lambda = \frac{\lambda_1 + \lambda_2 + \lambda_3 + --- + \lambda_n}{N} \quad (a)$$

$$\boxed{\lambda = \frac{\bar{v}\, t}{N}}$$

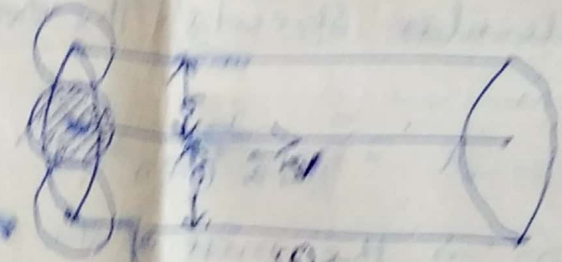
where $\bar{V}$ is the average speed of the molecule.

Expression →

Considering a gas possessing n - molecules per unit volume. It is assumed that only one molecule which is under consideration is in motion while all the others are at rest. If σ is the diameter of each molecule, then the moving molecule will collide with all those molecules, whose centres lie within a distance σ from its centre as shown in fig (b).



(b)



(c)

If v is the velocity of the moving molecule, then in one second it will collide with all the molecules whose centres lie in a cylinder of radius σ and length v i.e within a volume $\pi\sigma^2 v$.

The no. of collisions made by the moving molecule in one second is given by

$$N = \pi\sigma^2 v n$$

[Note — The no. of molecules present in a volume $\pi\sigma^2 v$ $= \pi\sigma^2 v n$]

where n is the no. of molecules per unit volume.

As the distance traversed by the molecule in one sec is v. therefore, the mean path is given by

$$\lambda = \frac{\text{Total distance traversed in one sec}}{\text{No. of Collision suffered by the molecules in one sec.}}$$

$$= \frac{v}{\pi\sigma^2 v n} \quad \text{——— (1)}$$

$$= \frac{1}{\pi\sigma^2 n}$$

This expression has been derived by assuming that only one molecule is moving while all others are at rest which does not represent the state of affairs. Therefore, Maxwell assumed the realistic assumption that all the molecules are moving with all possible velocities in all possible directions and found the result by applying the distribution law of molecular speeds. The exact result obtained by him is

$$\lambda = \frac{1}{\sqrt{2}\,(\pi\sigma^2 n)} \quad \text{—— (2)}$$

$\rho = \frac{mn}{1}$ If m is the mass of the molecule, then $mn = \rho$ where ρ is the density of the gas

Thus expression for mean free path in terms of density becomes

$$\lambda = \frac{m}{\sqrt{2}\,(\pi\sigma^2\rho)} \quad \text{—— (3)}$$

$$\lambda = \frac{m}{\sqrt{2}\,(\pi\sigma^2\rho)} \quad \text{—— (3)}$$

The mean free path is inversely proportional to the density of the gas.

According to Kinetic theory of gases, the pressure of the gas is given by

$$P = \frac{1}{3}\rho\bar{v}^2$$

where $\bar{v}^2$ is the mean square speed of the molecules.

$$\bar{v}^2 = \frac{3KT}{m}$$

$$P = \frac{1}{\cancel{3}}\rho.\frac{\cancel{3}KT}{m}$$

$$\frac{m}{\rho} = \frac{KT}{P} \quad \text{—— (4)}$$

Putting (4) in (3)

$$\boxed{\lambda = \frac{KT}{\sqrt{2}\,(\pi \sigma^2 P)}}$$

This is the expression for the mean free path in terms of absolute temperature and pressure of the gas. According to this relation, the mean free path is directly proportional to the absolute temperature of the gas and inversely proportional to pressure of the gas.
